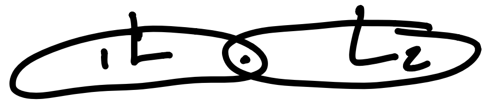
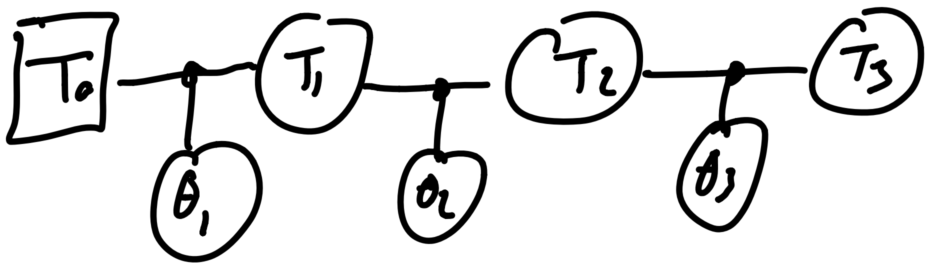
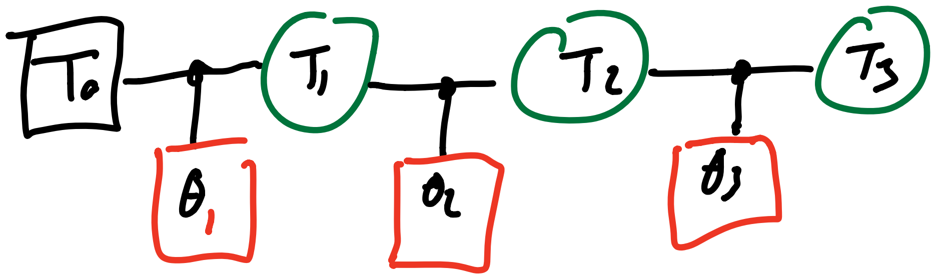
KINEMATICS

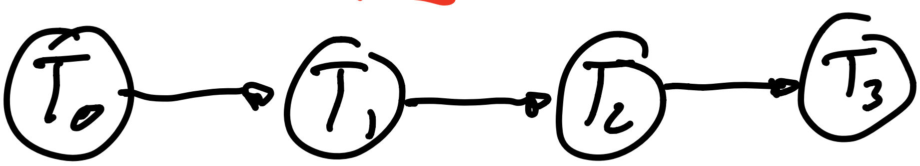
$_1T_2(\theta) = {}_1T_j \, e^{Z\theta} \, {}_jT_2$





FK: easy



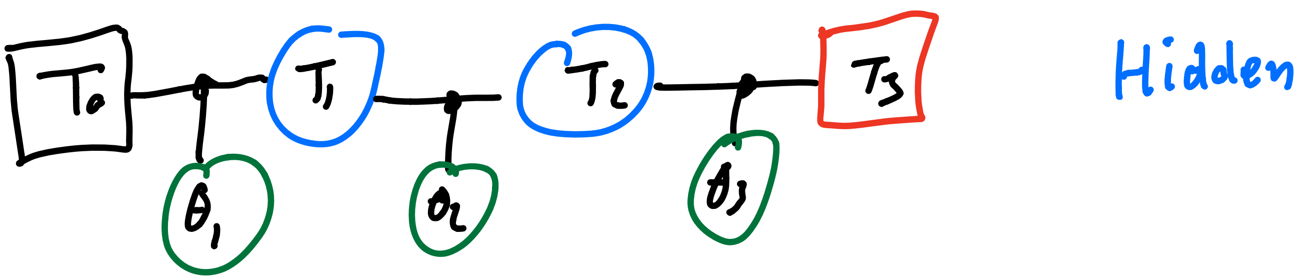


$T_0 =$

$T_1 = T_0 \, {}_0T_1(\theta_1)$

$T_3 = T_2 \, {}_2T_3(\theta_3)$

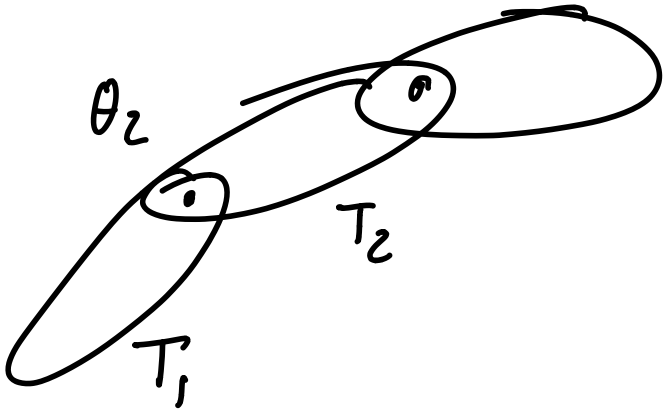
IK: non-linear :-(→ multiple solutions
IKFAST



$|\theta| < |T_3|$ → underactuated, add T_3 factor

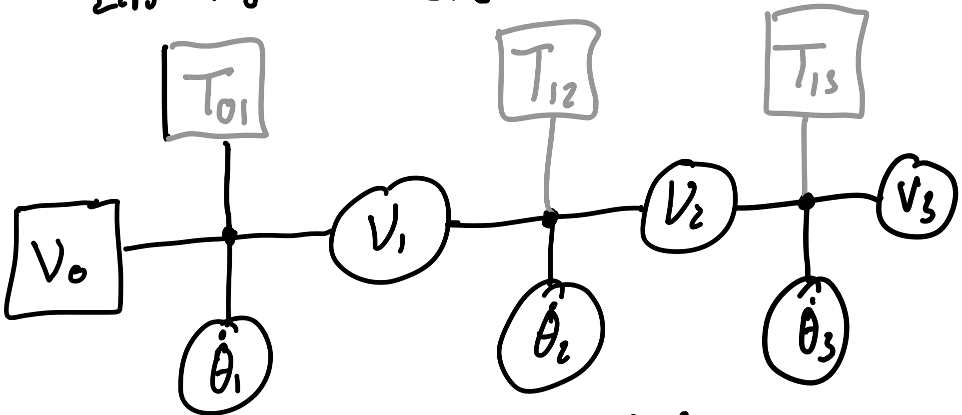
$|\theta| > |T_3|$ → overactuated, add θ factor

DIFFERENTIAL KINEMATICS

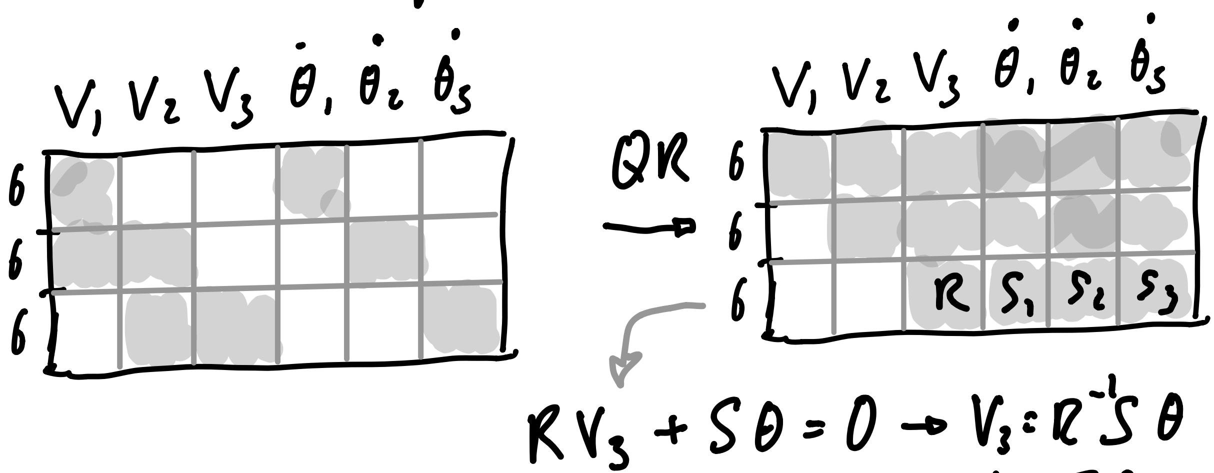


$$V_2 = Ad_{T_{21}} V_1 + A_2 \dot{\theta}_2$$ LINEAR!

$$\begin{bmatrix} R & \\ [p]R & R \end{bmatrix} \qquad \begin{bmatrix} \bar{\omega} \\ \bar{p} \end{bmatrix}$$



FG $\Longleftrightarrow$ Matrix



$$RV_3 + S\theta = 0 \rightarrow V_3 = R^{-1} S \theta$$

$$V_3 = J\theta$$

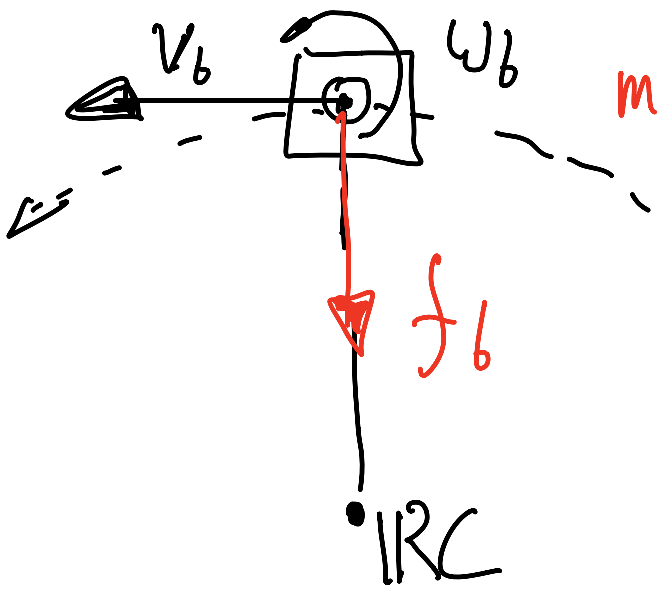
$$\Theta \xrightarrow{J} V_3$$

Dynamics

(8.22) $$f_b = m\dot{v}_b + \omega_b \times m v_b$$ FORCE

(8.23) $$\underbrace{m_b = I_b \dot{\omega}_b}_{F = ma} + \underbrace{\omega_b \times I_b \omega_b}_{\text{coriolis}}$$ MOMEN

Constant twist:

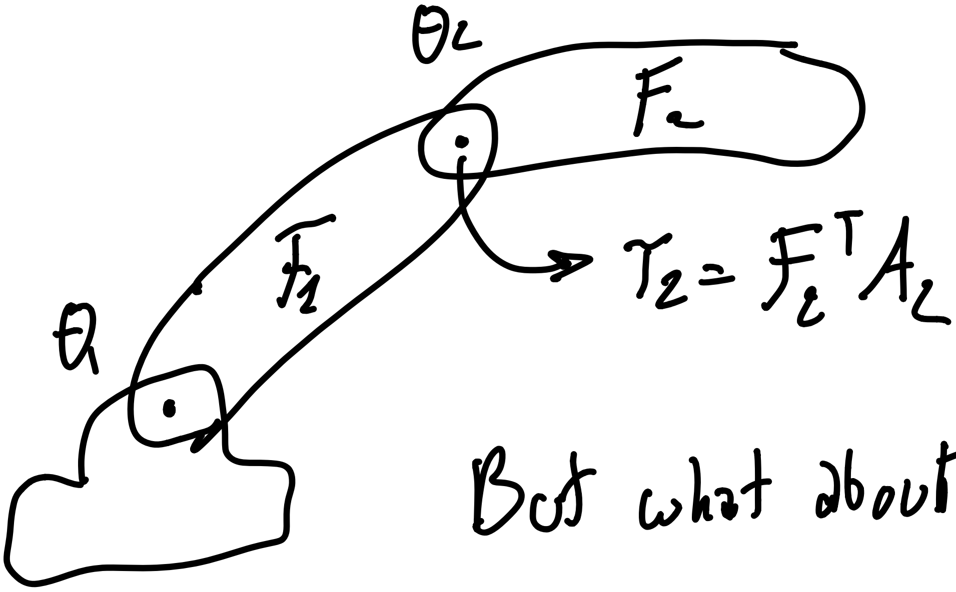


$$m_b = \underbrace{\omega_b \times I_b \omega_b}_{\text{zero for sphere, cube if } \{b\} \text{ aligned.}}$$

$$\mathcal{F}_b = \begin{bmatrix} m_b \\ f_b \end{bmatrix} = \begin{bmatrix} I_b & \\ & mI \end{bmatrix} \begin{bmatrix} \dot{\omega}_b \\ \dot{v}_b \end{bmatrix} - \begin{bmatrix} \hat{\omega}_b & \\ \hat{v}_b & \hat{\omega}_b \end{bmatrix}^T \begin{bmatrix} I_b & \\ & mI \end{bmatrix} \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}$$

$$\underbrace{\mathcal{F}_b = \mathcal{G}_b \dot{\mathcal{V}}_b}_{F = ma} \underbrace{- [\mathrm{ad}_{\mathcal{V}_b}]^T \mathcal{G}_b \mathcal{V}_b}_{\text{coriolis}}$$

$$F_1 - Ad_{T_{21}}^T F_2 = \mathcal{G}_1 \dot{\mathcal{V}}_1 - [ad_{\mathcal{V}_1}]^T \mathcal{G}_1 \mathcal{V}_1$$

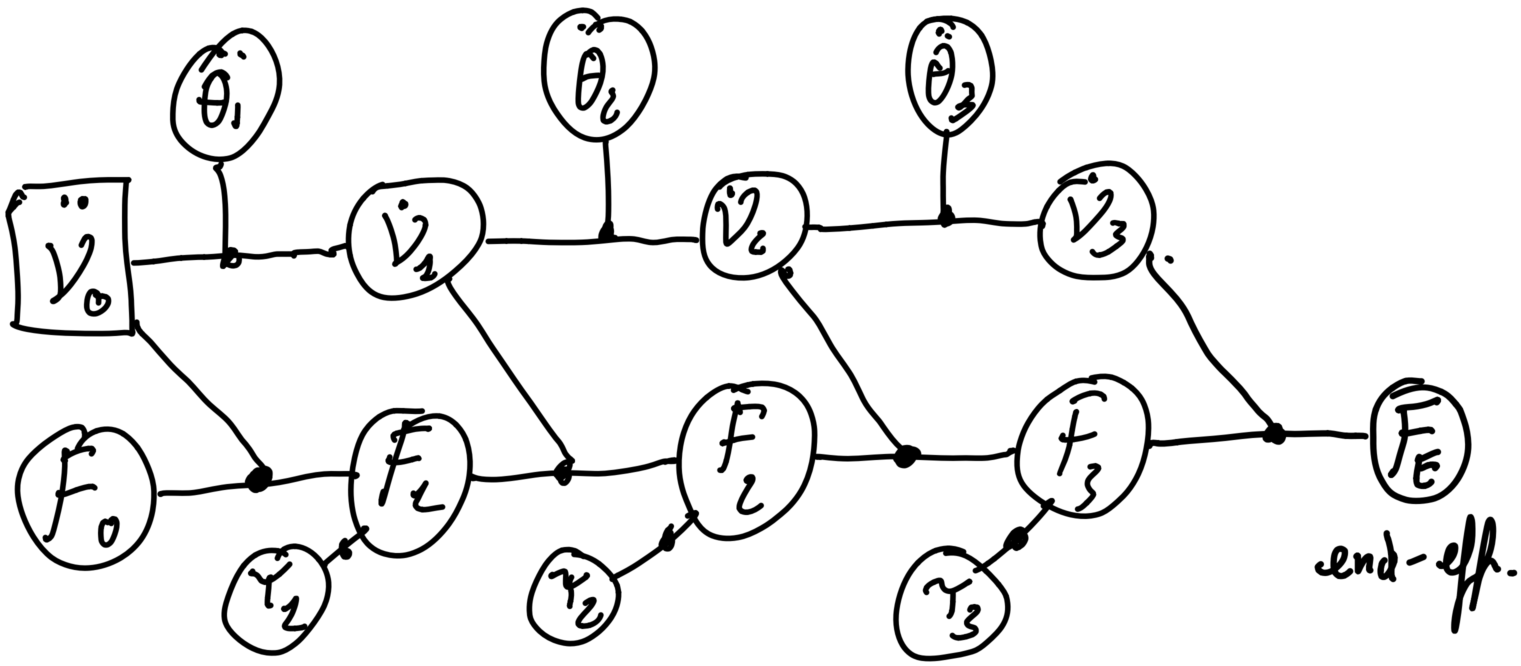


But what about $\dot{\mathcal{V}}_1$?

Review: $\mathcal{V}_2 = Ad_{T_{21}} \mathcal{V}_1 + A_2 \dot{\theta}_2$

$\xrightarrow{\text{diff}}$ $\dot{\mathcal{V}}_2 = Ad_{T_{21}} \dot{\mathcal{V}}_1 + A_2 \ddot{\theta}_2 + [ad_{\mathcal{V}_2}] A_2 \dot{\theta}_2$

$\Rightarrow$ Fix $\theta, \dot{\theta}, \mathcal{V}, \tau$: $\boxed{\text{LINEAR!}}$



Forward : $\tau \rightsquigarrow \ddot{\theta}$ $\ddot{\theta} \rightarrow \dot{\mathcal{V}}_3$

Inverse : $\ddot{\theta} \rightsquigarrow \tau$ $\dot{\mathcal{V}}_3 \xrightarrow{?} \ddot{\theta}$

similarly hard

Forward iterations Given $\theta, \dot{\theta}, \ddot{\theta}$, for $i = 1$ to n do

$$T_{i,i-1} = e^{-[\mathcal{A}_i]\theta_i} M_{i,i-1}, \tag{8.50}$$ FK

$$\mathcal{V}_i = \mathrm{Ad}_{T_{i,i-1}}(\mathcal{V}_{i-1}) + \mathcal{A}_i \dot{\theta}_i, \tag{8.51}$$ FK

$$\dot{\mathcal{V}}_i = \mathrm{Ad}_{T_{i,i-1}}(\dot{\mathcal{V}}_{i-1}) + \mathrm{ad}_{\mathcal{V}_i}(\mathcal{A}_i)\dot{\theta}_i + \mathcal{A}_i \ddot{\theta}_i. \tag{8.52}$$ ACC

Backward iterations For $i = n$ to 1 do

$$\mathcal{F}_i = \mathrm{Ad}^{\mathrm{T}}_{T_{i+1,i}}(\mathcal{F}_{i+1}) + \mathcal{G}_i \dot{\mathcal{V}}_i - \mathrm{ad}^{\mathrm{T}}_{\mathcal{V}_i}(\mathcal{G}_i \mathcal{V}_i), \tag{8.53}$$ ω

$$\tau_i = \mathcal{F}_i^{\mathrm{T}} \mathcal{A}_i. \tag{8.54}$$ τ